

Language: FOL + Primitive binary relations: "is a part of" denoted by p and "is collected by" denoted by $@$.

Axiom: $(\forall x. x p y \Rightarrow x p z) \Rightarrow y p z$

Axiom: $x p y \wedge y p z \Rightarrow x p z$

Define (=): $x=y \Leftrightarrow x p y \wedge y p x$

Define (pp): $x p p y \Leftrightarrow x p y \wedge \neg y p x$

Axiom: $x @ y \wedge z p x \Rightarrow z @ y$

Define (unit): $\text{unit}(x) \Leftrightarrow (\exists c. \forall y. y @ c \Leftrightarrow y p x) \wedge (\neg \exists c, z. z p p x \wedge \forall y. y @ c \Leftrightarrow y p z)$

Axiom: $\text{unit}(x) \wedge \text{unit}(y) \wedge x \neq y \Rightarrow \neg \exists z. z p x \wedge z p y$

A particle is a proper part of a unit.

Define (collection): $\text{collection}(x) \Leftrightarrow (\forall y. y p x \Rightarrow \exists c, z. z p y \wedge z p c \wedge \text{unit}(c) \wedge c p x)$

Define (e): $x e y \Leftrightarrow \text{unit}(x) \wedge x p y \wedge \text{collection}(y)$

e is read as: is a trivial member of.

Axiom schema: if ϕ is a formula in which x is not free, then $(\exists z. \text{unit}(z)\phi \Rightarrow \exists x. \forall y. y e x \Leftrightarrow \text{unit}(y)\phi)$ is an axiom.

Define [[]]: $y=[x|\phi] \Leftrightarrow \text{collection}(y) \wedge (\forall x. x e y \Leftrightarrow \text{unit}(x)\phi)$

Define (precollector): $\text{precollector}(x) \Leftrightarrow \exists y. y @ x$

Define (collector): $\text{collector}(x) \Leftrightarrow \text{unit}(x) \wedge \exists y. y @ x$

A proper precollector is a precollector that is not a unit.

A class is a collection of collectors where distinct collectors do collect distinct collections.

Define (class): $\text{class}(x) \Leftrightarrow \text{collection}(x) \wedge$
 $(\forall y. y \in x \Rightarrow \text{collector}(y)) \wedge (\forall y, z. y \in x \wedge z \in x \wedge y \neq z \Rightarrow$
 $\exists u, w. u = [k \mid k @ y] \wedge w = [k \mid k @ z] \wedge u \neq w)$

For every collector x the collection $[y \mid y @ x]$ is called the *extension* of x , and x is its *exclusive* collector.

When this extension is a class, it's called a *class extension* of x .

A set is a class extension of a collector.

Epsilon membership is defined as:

$x \in y \Leftrightarrow \text{class}(y) \wedge \exists z. \text{collector}(z) \wedge x = [u \mid u @ z] \wedge z \in y.$

This explains those terms in standard set\class theories.

However in set theories it is sufficient to interpret sets as collectors and epsilon membership as "is a unit collected by the collector".

Zuhair Al-Johar, 21/5/2011