The following is a trial to give some informal followed by formal explanation as to what various objects that known set\class theories speaks about are.

Informal account: The process of "collecting objects" is imagined here as: putting objects into a container by a collector.

The result of the collective action of a collector is a collection. However that doesn't mean that every collection is the result of the collective action of a collector.

A collection is defined as an aggregate of atoms. Atoms are indivisible objects, i.e. do not have proper parts of them. An element (member) of a collection is an atom of that collection.

A collector of at least one atom is extended by the collection of all atoms it collects; this collection is called as the *extension* of it.

An Ur-element is an atom extending a collector that is not a collector. A class is a collection of collectors where different collectors have different extensions. When the extension of a collector is a class, it's called a *class extension* of it. A class extension of a collector is a *set*; a class that is not an extension of a collector is a *proper class*. Epsilon membership "$\in$" is defined as: "is an extension of a collector that is a member of a class". Alternatively one can simply define sets as collectors, and $\in$ as "is an atom collected by", and this would be sufficient for interpreting $\in$ in ZF.

Formal Account:

Language: First order logic

Primitive binary relations[1]: "p" standing for part; "in" standing for "is contained in"; "O" standing for "owns".

Axiom1: $\forall x,y. (\forall z.\ z\ p\ x \Rightarrow z\ p\ y) \Rightarrow x\ p\ y$

Axiom2: $\forall x,y,z. (x\ p\ y \wedge y\ p\ z \Rightarrow x\ p\ z)$

Define (=): $x=y \Leftrightarrow x\ p\ y \wedge y\ p\ x$

Define (pp): $x\ pp\ y \Leftrightarrow x\ p\ y \wedge \neg\ y\ p\ x$

Define (atom): $atom(x) \Leftrightarrow \neg\exists y.\ y\ pp\ x$

Define (collection):

$collection(x) \Leftrightarrow (\forall y.\ y\ p\ x \Rightarrow \exists z.\ atom(z) \wedge z\ p\ y)$

Axiom3: $\forall x.\ collection(x)$.

Define (e): $x\ e\ y \Leftrightarrow x\ p\ y \wedge atom(x)$

e read as: is a member of.

Axiom scheme 4: if ø is a formula in which x is not free, then $(\exists z.atom(z)ø \Rightarrow \exists x\forall y.(y\ e\ x \Leftrightarrow atom(y)\ ø))$ is an axiom.

Define ([|]): $y = [x|ø] \Leftrightarrow (\forall x.\ x\ e\ y \Leftrightarrow atom(x)ø)$

Define (collector): $collector(x) \Leftrightarrow atom(x) \wedge \exists c.\ x\ O\ c$

Define (collected by): y collected by x $\Leftrightarrow$

$collector(x) \wedge \exists c.(x\ O\ c \wedge \forall z.\ z\ e\ y \Rightarrow z\ in\ c)$

[1]The last two primitives can be replaced by one primitive relation *collected by* where collectors are defined as atoms other than Ur-elements, and Ur-elements are left to be treated according to the relevant theory.

Classes and Sets: For every collector x collecting at least an atom, there is a collection x* which extends it, this is defined as: x*= [y| y collected by x], x* is called the **extension** of x, and x is the **exclusive collector** of x*.

An **Ur-element** is an atom extending a collector that is not a collector.

Define(Ur): $Ur(x) \Leftrightarrow atom(x) \land \exists z.collector(z) \land x=z^* \land \neg \exists c.x \text{ O } c$

A **class** is a collection of collectors where different collectors *have* different extensions.

For any collector x when x* is a class it is said to be the **class extension** of x.

A **proper class** is a class that is not an extension of a collector; *even if it is collected by a collector!*

Sets are defined as class extensions of collectors.

Epsilon membership is defined as:

Define($\in$): $x \in y \Leftrightarrow class(y) \land \exists z.\ collector(z) \land x=z^* \land z \text{ e } y$

These definitions explain those terms in any set\class theory. However in Standard set theories like ZF were all objects are sets; the easiest way to define a set is as a collector; and epsilon membership as: "is an atom collected by", and with Axiom schema 4 one can define proper classes enabling us to define theories equivalent to NBG and MK.

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